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Constructing null networks for community detection in complex networks

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Abstract. Communities are virtually ubiquitous in real-world networks, and the statistic of modularity index Q is the classical measurement for community detection algorithms. However, the relationship between the modularity property and network multilever micro-scale structures is still not clear. In this paper, we study community detection results both in artificial and real-life complex networks by constructing different order null networks, and the results uncover that how micro-structures (such as degree distribution, assortativity and clustering coefficient) affect community properties. Meanwhile, we also propose two novel null networks (increasing or decreasing community structures) to verify the robustness of different community detection algorithms. Our results indicate that the modularity index Q is not a suitable statistic to measure the weak community property which is widely available in empirical networks. Our findings can not only be used to test the robustness of different community detection methods, but also be helpful to uncover the correlation of network structures between microcosmic and mesoscopic scales.

1 Introduction

Community structures are virtually ubiquitous in many real-world complex networks, such as protein-protein interaction network [1,2], graph of the World Wide Web [3,4], metabolic network [5], and food web network [6,7]. Therefore, studies of network communities have been a research hotspot in recent years [8–10]. Many previous studies are aiming at detecting and measuring the modularity of community structures in complex networks [11,12]. To get a high modularity performance, a variety of detection algorithms have been proposed, such as MultiLevel algorithm [13], the greedy optimization of modularity algorithm (CNM algorithm) [14], WalkTrap algorithm [15], the algorithm based on edge betweenness (GN algorithm) [16], KClique detection algorithm [17], InfoMap algorithm [18], the algorithm based on community leading eigenvector (LPLS algorithm) [19], the algorithm based on label propagation [20] and the Differential Evolution based Community Detection algorithm (DECD algorithm) [21,22].

Meanwhile, the term of null model had been explicitly proposed by Colwell et al. in a conference [23]. Generally, a random network with some of the same properties as the real-life network, is called a null network of the original network [24,25]. Null networks can provide an accurate reference for the original network, and can accurately describe the non-trivial characteristics of the original network combined with the statistic indicators, which can help us to reveal the origin and the level of complexity. Null networks have been widely applied in analyzing clustering coefficient [26,27], degree distribution [28] and link prediction [29,30].

In existing researches, the relationship between micro-scale structures (e.g. degree distribution, assortativity and clustering coefficient) and meso-scale characteristics (i.e., community properties) is not very clear. For example, the modularity index Q [14] can measure the quality of community detection algorithms by comparing the original network and its corresponding $1k$ null network ($1k$ means the order of null networks) [31]. But $1k$ null network only keeps the degree distribution of the original network instead of keeping the other micro-structures. We do not know how the $2k$–$3k$ micro-scale structures affect community properties. Meanwhile, if a network shows a strong community structure, the edges of this network can be divided into two types: the edges within a community and the edges between communities. Actually, we do not know how the two types of edges affect the community structure of the entire network. Furthermore, the impact of different community intensities on community detection algorithms is not clear.

To address the above mentioned problems, first of all, we maintain the micro-scale structures at multi levers by

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constructing $1k$–$3k$ null networks to study how micro-structures affect community properties. Experimental results show that $1k$–$2k$ micro-structures (degree distribution and assortativity) are not enough to keep meso-scale characteristics, but clustering coefficient (transitivity) is a very significant micro-scale structure for maintaining meso-scale characteristics of the original network. Then, we propose two novel null networks, which can maintain two types of link relationships respectively (edges within a community or edges between communities) of the original network. In the situation of keeping meso-scale characteristics of the original network, we study how the two types of edges affect community structures of the entire network. Our results suggest that keeping community properties of the original network but not keeping its micro-structures are enough for community detection. Finally, we propose another two novel null networks, namely increasing and decreasing community structures to verify the performances of different community detection algorithms. We find that the detection algorithms show remarkable difference for detecting community structures of low intensities. Furthermore, our results imply that the modularity index Q is not a suitable statistic for the network of the weak community property. In conclusion, our findings can not only be used to test the robustness of different community detection algorithms, but also be helpful to uncover the correlation relationship of network structures between micro-scale and meso-scale.

The rest of our paper is the following. In Section 2, we study how micro-scale structures affect community properties. In Section 3, we uncover the impact of the two types of edges on community structures. In Section 4, we study the impact of community structures of different intensities on community detection algorithms. Section 5 is the conclusion of this study.

2 The impact of micro-scale structures on meso-scale characteristics

2.1 Different order null networks

In previous studies, scholars not only paid attention to the absolute values of the original network statistics, but also more concerned about the relative values of the same statistic after comparing the original network and the randomization null networks [32,33]. Generally, a random network with some of the same properties of the original network is called a null network [24,25]. The main purpose of using null networks is to detect the non-trivial characteristics of the original network, which requires a gradual approximation of the nature of the original network from rough to precise. In order to gradually approach the original network, Mahadeven et al. presented a new, systematic approach for analyzing network topologies, and they introduced the dK-series of probability distributions specifying all degree correlations within d-sized subgraphs of a given graph G [31,34]. Orsini et al. employed the dK-series to study the statistical dependencies between different network properties [35].

$0k$–$3k$ null networks are the most commonly used null networks in previous researches. All order null networks are interrelated, that is, $0k \supseteq 1k \supseteq 2k \cdots\cdots (n-1)k \supseteq nk$. Any nk null network will contain the characteristics of $(n-1)k$ null network. A summary of different order null networks ($0k$–$3k$) is shown in Figure 1. All null networks are generated by a random rewiring algorithm, which can randomize some kinds of edges [36,37]. Figure 1a illustrates the construction of the properties Pd, which we will call the dK-series (The $d = 0, \cdots, 4$ of Pd corresponds to $k = 0, \ldots, 4$ of dK-series). We used the total numbers of corresponding subgraphs represent the values of all distributions P, i.e., $P(2, 2) = 2$ means that G contains 2 edges between two 2-degree nodes. $0k$ null network is the simplest and the most randomized null model, which only keeps the number of nodes and the average node degree with the original network. The rewiring process of $0k$ null network is illustrated in Figure 1b. $1k$ null network can maintain the degree distribution of the original network to random rewire edges. The rewiring process of $1k$ null network is illustrated in Figure 1c. $2k$ null network and the original network possess the same joint degree distribution, which means they have the same degree values for the nodes on the ends of each link. The rewiring process of $2k$ null network is illustrated in Figure 1d. $2.5k$ null network and the original network possess the same joint degree distribution and the degree-dependent average clustering coefficient. $3k$ null network and the original network possess the same clustering coefficient of all nodes. The rewiring process of $2.5k$ (or $3k$) null network is illustrated in Figure 1e.

In order to show different order null networks more clearly, we used Figure 2 to illustrate the network constructed by real HOT topology data [38] and its corresponding $0k$, $1k$, $2k$, 2.5 and $3k$ null networks. Figure 2a is the original Hot network graph and Figure 2b is its $0k$ null network graphs. Figure 2c is its $1k$ null network graphs and Figure 2d is its $2k$ null network graphs. Figure 2e is its $2.5k$ null network graphs and Figure 2f is its $3k$ null network graphs. We found that with the increasing of null model order, the null networks are gradually approaching the original HOT network.

2.2 The modularity index Q of null networks

Community modularity Q is used to measure the quality of community detection algorithms [14]. The definition of modularity is as follows:

$$\mathrm{Q} = \frac{1}{2M} \sum_{i,j} \left(A_{ij} - \frac{k_i k_j}{2M} \right) \delta(C_i, C_j), \qquad (1)$$

where M is the total number of edges in network; $A_{ij} = (a_{ij})_{n\times n}$ is the adjacency matrix; k_i and k_j are the degrees of nodes i and j; $\delta(C_i, C_j)$ indicates the community relationship. If nodes i and j belong to the same community, the value of $\delta(C_i, C_j)$ is 1; or else the value is 0. A higher modularity index Q usually means a better result of community detection. Usually, we get the

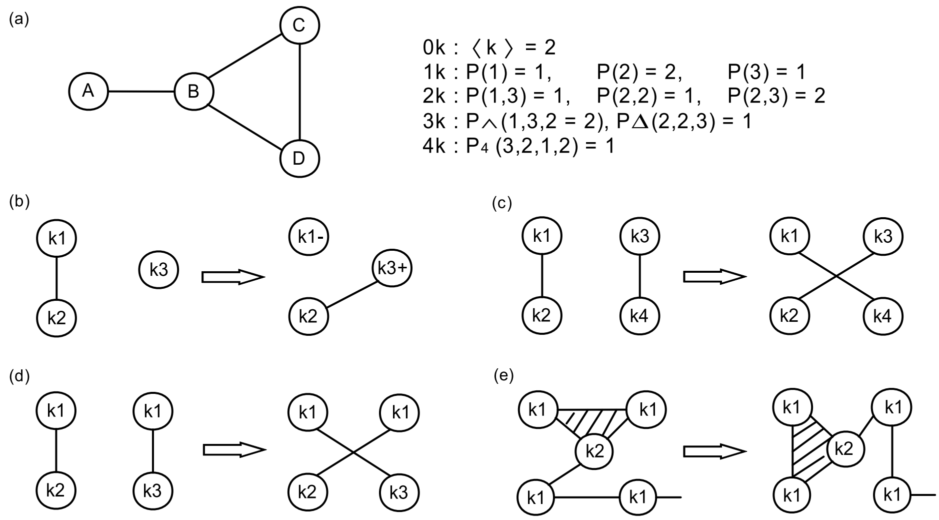

Fig. 1. The summary of $0k$–$3k$ null models. (a) The properties Pd, $d = 0, \ldots, 4$, calculated for a given graph G of size 4, (b) the rewiring process of $0k$ null network, (c) the rewiring process of $1k$ null network, (d) the rewiring process of $2k$ null network, (e) the rewiring process of $2.5k$ and $3k$ null networks.

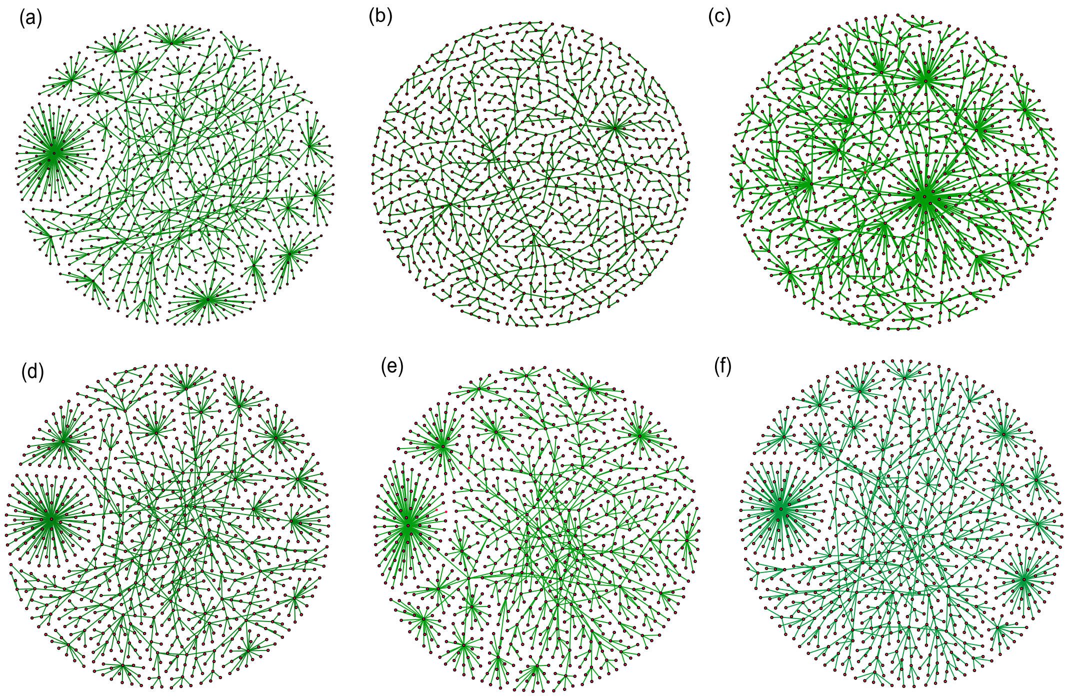

Fig. 2. The original Hot network graph and its corresponding null network graphs. (a) The original Hot network graph, (b) its $0k$ null network, (c) its $1k$ null network graphs, (d) its $2k$ null network, (e) its $2.5k$ null network, and (f) its $3k$ null network.

Q value only by comparing the original network with its corresponding $1k$ null network. However, $1k$ null network only keeps the degree distribution of the original network instead of keeping high order micro-scale structures (e.g., assortativity and clustering coefficient). We do not know how $2k$–$3k$ micro-structures affect community properties. Hence, we generate higher order ($2k$–$3k$) null networks to study how micro-structures affect community properties.

In this study, we employ two open and commonly used data sets as our experiment networks (Zachary's karate club network [39] and Lusseau's dolphin network [40]). Here, we repeat our simulations 100 times independently to achieve the average result. The karate club network was built by Zachary when they studied the relationships among the members of karate club. In this network, nodes represent the club members and edges represent the social relationships between the members. The dolphin social network is an undirected social network of frequent associations between 62 dolphins in a community living off Doubtful Sound, New Zealand.

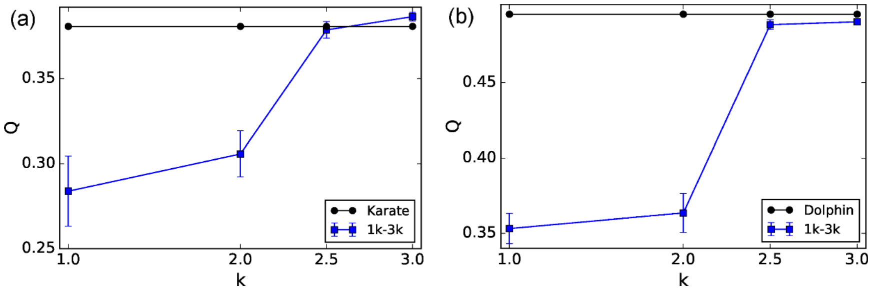

Fig. 3. The modularity Q values of different order null networks ($1k$–$3k$). The x-coordinate represents the order of null networks, and the y-coordinate represents modularity Q values. We employ the CNM algorithm to get the modularity Q values. The dot line represents the original network. (a) The modularity Q results of Zachary's karate network. (b) The modularity Q results of Lusseau's dolphin network.

To study the impact of micro-scale structures on meso-scale characteristics. Firstly, we employ CNM algorithm to get the modularity Q values of the karate and dolphin networks. Secondly, we maintain the micro-scale structures at multi levers by constructing $1k$–$3k$ null networks. Thirdly, we use CNM algorithm to get the modularity Q values of null networks constructed in step 2.

As shown in Figure 3, the modularity Q of $1k$ null network is larger than 0, but lesser than that of the original network. This phenomenon shows that the structure of the original network is very different from its $1k$ null network. Compared with $1k$ null network, the Q value of $2k$ null network is closer to the original network, because $2k$ null network maintains more micro-structures of the original network (i.e., assortativity). However, there is still a large difference for the Q value between the original network and its $2k$ null network. Furthermore, the modularity Q value of $2.5k$ and $3k$ null networks are very similar to that of the original network, because $2.5k$ and $3k$ null networks keep the clustering characteristic of the original network. Hence, the above results suggest that the $1k$–$2k$ micro-scale structures (degree distribution and assortativity) are not enough to keep meso-scale characteristics (i.e., community properties), but $3k$ micro-scale structure (clustering coefficient) is a very significant micro-scale structure for maintaining the meso-scale characteristics of the original network.

3 The impact of two types of edges on community properties

3.1 Null network of random rewiring edges within a community

Traditional null networks are based on random rewiring, which completely destroy community structures of a network. The community structure characteristic is that the density of inner edges is comparatively higher than the density of external edges between communities. To study the impact of inner edges on community structures, we introduce the first null network, which only changes the inner structure of each community but maintains the number of communities and the community structure characteristics.

The construction process of the null network of random rewiring edges within a community is shown in Figure 4. Firstly, we divide the original network into multiple communities by a classical community detection algorithm. Secondly, we only exchange edges within a community, in the case of keeping community structures unchanged. For example, we disconnect the two edges $A1$–$A4$ and $A2$–$A3$ in community-A. If the pairs of nodes $A1$–$A3$ and $A2$–$A4$ are not connected, we connect them. The result of the reconnection is shown in Figure 4b. The process of random rewiring is repeated many times until getting a completely randomized version of the null network. For generating the null networks of rewiring edges within a community, we can get the $1k$–$3k$ null networks that only destroy the inner structure of each community. Moreover, we can use the above null networks to change the structure of each community independently, and to study the impact of each community on the entire network.

3.2 Null network of random rewiring edges between communities

To study the impact of external edges on community structures, we introduce the second null network, which only changes the links between two communities but maintains the number of communities and the structure characteristics within each community. The construction process of the null network of random rewiring edges between communities is shown in Figure 5. Firstly, we divide the original network into multiple communities by a classical community detection algorithm. Secondly, we only exchange edges between communities, in the case of keeping other structures unchanged. For example, we disconnect the two edges $A1$–$B1$ and $A4$–$B4$. If the pairs of nodes $A1$–$B4$ and $A4$–$B1$ are not connected, we connect them. The result of the reconnection is shown in Figure 5b. The process of random rewiring is repeated many times until getting a completely randomized version of the null network. In addition, we can also get the $1k$–$3k$ null networks that only destroy the external structure of all communities. Our two null networks that are depicted in Figures 4 and 5 can distinguish the role of edges within a community and that of edges between communities for the entire network.

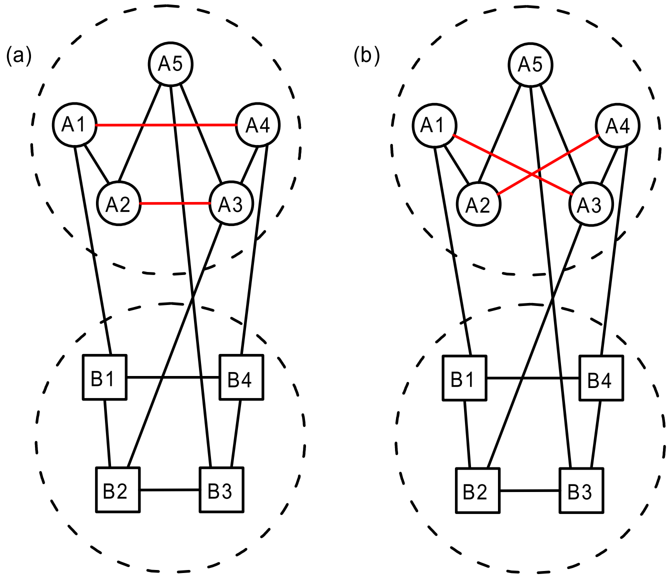

Fig. 4. The construction process of the null network of random rewiring edges within a community. We remove the links $A1$–$A4$ and $A2$–$A3$, and then add the links $A1$–$A3$ and $A2$–$A4$. (a) The original toy model, and (b) its null network of random rewiring edges within a community.

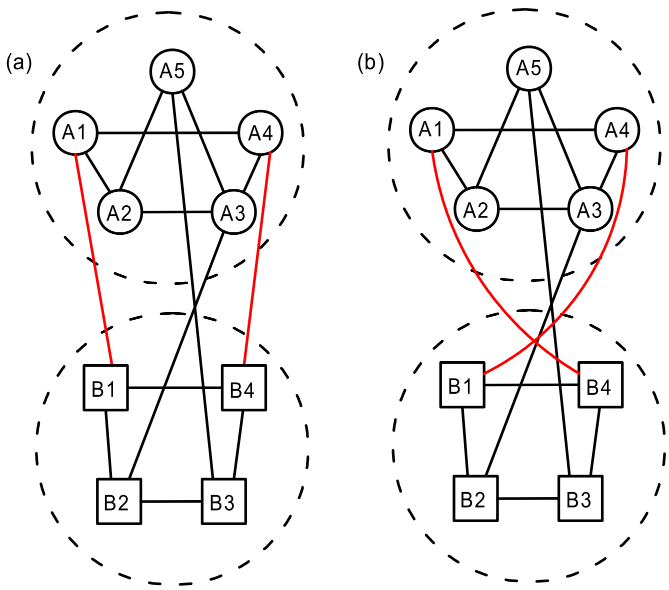

Fig. 5. The construction process of the null network of random rewiring edges between communities. We remove the links $A1$–$B1$ and $A4$–$B4$, and then add the links $A1$–$B4$ and $A4$–$B1$. (a) The original toy model, and (b) its null network of random rewiring edges between communities.

3.3 The impact of two types of edges

The existing $1k$–$3k$ null networks not only change the micro-structures of the original network, but also destroy the meso-scale characteristics of the original network. But our two novel null networks do not change the community structure characteristics of the original network. Furthermore, we use the community structure to divide the edges into two types: the edges in a community and the edges between two communities. In this chapter, we use the proposed null networks to study how the two types of edges affect community structures of the entire network.

The modularity Q results of the two proposed null networks are shown in Figure 6. We find that the Q values of the two kinds of null networks are closer to that of the original network than traditional $1k$–$3k$ null networks. The results suggest that rewiring the two types of edges have a small impact on community structures, in the case of maintaining the community structure characteristics. Our finding implies that keeping community properties of the original network but not keeping its micro-structures are enough for community detection.

3.4 The impact of a single community structure

If we divide a network into more communities than one, we can uncover the role of every community by constructing the null network of random rewiring edges within a community. For example, firstly, we divide the karate network into three communities by CNM algorithm in Figure 7a. The first community contains 8 nodes and 12 edges, the second community contains 9 nodes and 13 edges and the last community contains 17 nodes and 34 edges. Secondly, we construct the null networks of rewiring edges within each community respectively, in the case of keeping other structures unchanged. Thirdly, we re-calculate the modularity Q of the null networks that constructed in step 2.

The results of manipulating each community have been shown in Figure 7b. The Q value of the original network is 0.381. After rewiring edges within community 1, the modularity Q value of the entire network is 0.387. If we apply the same method to the community 2 and community 3, the new modularity Q values of the entire network are 0.381 and 0.384 respectively. The above results suggest that the inner topology structures of three communities have different roles for community properties of the entire network. Especially, community 1 and community 3 show a more important role than that of community 2.

4 Measuring the robustness of community detection algorithms

4.1 Null network of increasing community structures

In general, given a network, whose community structure is settled, we want to study the impact of different community characteristics on community detection algorithms. Traditionally, we can only construct a series of benchmark network models [16,41]. However, the benchmark networks can not keep all the important statistics of the original network. To study the impact of increasing community structures on community detection, we introduce the third null network, which is enhances community structures of the original network. In the case of changing meso-scale characteristics, our null networks can

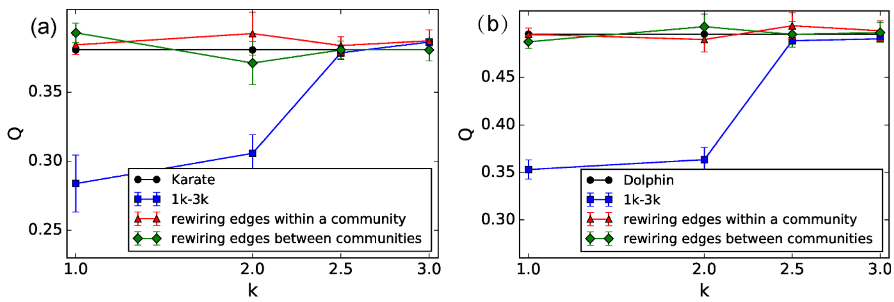

Fig. 6. The modularity results of rewiring edges by using the two proposed null networks. (a) The results of Zachary's karate network. (b) The results of Lusseau's dolphin network.

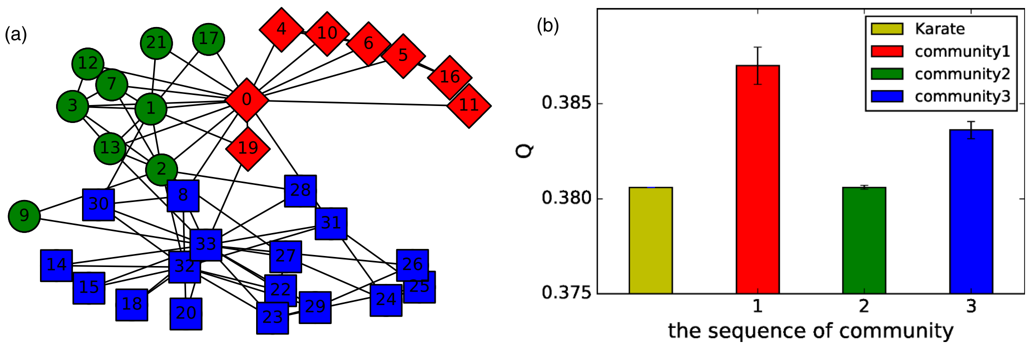

Fig. 7. The modularity Q of the null networks after rewiring edges within each community. (a) The karate network is divided by the CNM algorithm into three communities. (b) The modularity results after rewiring edges within each community by using $1k$ null network respectively.

maximally maintain micro-scale structures of the original network.

The construction process of the null network of increasing community structures is shown in Figure 8. Firstly, we divide the original network into multiple communities by a community detection algorithm. Secondly, we only exchange two external edges between two communities to be inner edges within a community, in the case of keeping other structures unchanged. For example, we disconnect the two edges $A1$–$B1$ and $A5$–$B3$. If the pairs of nodes $A1$–$A5$ and $B1$–$B3$ are not connected, we connect them. The result of the reconnection is shown in Figure 8b. The process of random rewiring is repeated many times according to the network scale until getting a completely randomized version of the null network. In addition, we also can get the $1k$–$3k$ null networks that enhance the community structures.

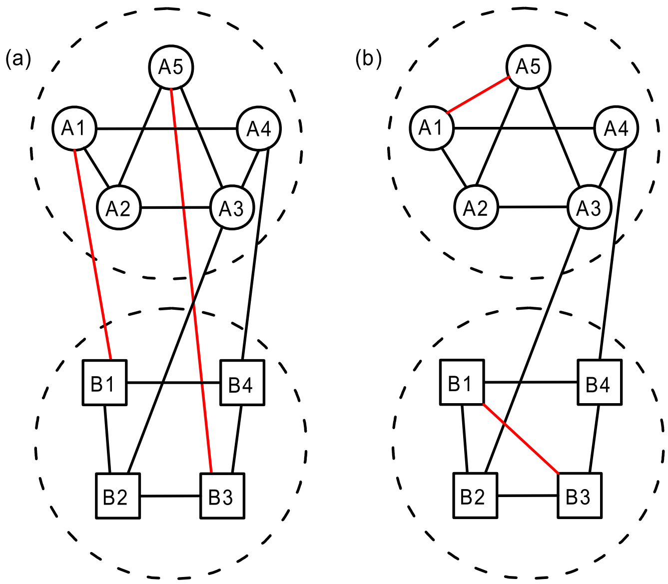

Fig. 8. The constructing process of the null network of increasing community structures. We remove the links $A1$–$B1$ and $A5$–$B3$, and then add the links $A1$–$A5$ and $B1$–$B3$. (a) The original toy model, and (b) its null network of increasing community structures.

4.2 Null network of decreasing community structures

To study the impact of decreasing community structures on community detection, we introduce the fourth null network, which weakens the community structures of the original network. According to our inference, the robustness of community detection algorithms is easier to be tested while the community structure is weak. The constructing process of the null network of decreasing community structures is shown in Figure 9. Firstly, we divide the original network into multiple communities by a community detection algorithm. Secondly, we only exchange two inner edges within a community to be external edges between two communities, in the case of keeping other structures unchanged. For example, we disconnect the two edges $A1$–$A4$ and $B1$–$B4$. If the

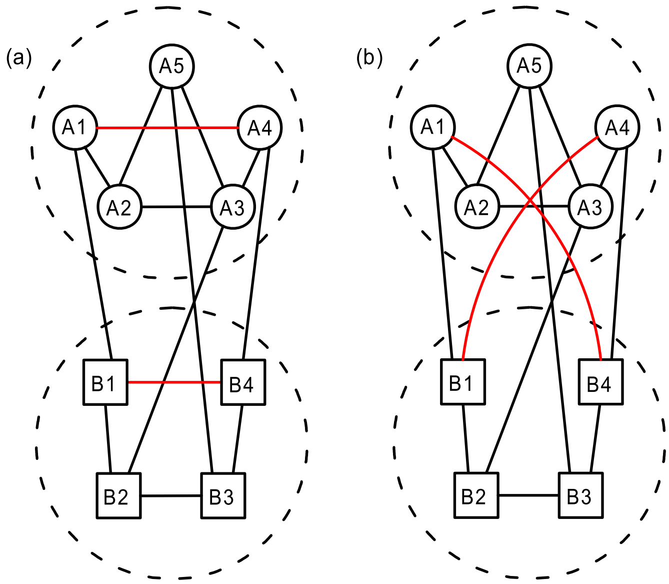

Fig. 9. The constructing process of the null network of decreasing community structures. We remove the links $A1$–$A4$ and $B1$–$B4$, and then add the links $A1$–$B4$ and $A4$–$B1$. (a) The original toy model, and (b) its null network of decreasing community structures.

pairs of nodes $A1$–$B4$ and $A4$–$B1$ are not connected, we connect them. The result of the reconnection is shown in Figure 9b. In addition, we also can get the $1k$–$3k$ null networks that weaken the community structures.

4.3 Normalized mutual information

The traditional modularity Q can measure the quality of community detection algorithms. In general, a higher modularity Q value usually means a better result of community detection. However, recently it is reported that Q has the problem of the resolution limit [42,43]. Normalized mutual information (NMI) is another index to measure the quality of the algorithms. Suppose that A and B are the real partition and predicted partition of the network respectively, we can define a confusion matrix C, where the rows correspond to the real communities defined in A, and the columns correspond to the predicted communities found in B. The element C_{ij} of C is the number of nodes appearing in the real community i and the predicted community j simultaneously. Based on the above definition of C, NMI is defined as follows [44]:

$$\mathrm{NMI}(A,B) = \frac{-2\sum_{i=1}^{C_A}\sum_{j=1}^{C_B} C_{ij}\log\left(\frac{C_{ij}N}{C_iC_j}\right)}{\sum_{i=1}^{C_A} C_i\log\left(\frac{C_i}{N}\right) + \sum_{j=1}^{C_B} C_j\log\left(\frac{C_j}{N}\right)}, \qquad (2)$$

where N is the total number of nodes in the network; C_A and C_B are the numbers of communities in partitions A and B; C_i is the sum over row i, representing the number of nodes in the ith community of partition A; C_j is the sum over column j, representing the number of nodes in the jth community of partition B. From the Equation (2), we can see that if A is equal to B, NMI takes its maximum value of 1. If B is completely different from A, NMI is equal to 0. NMI has the advantage of being able to accurately detect the quality of community detection in the context of knowing community structure. The disadvantage of NMI is that it is necessary to know the accurate information of community structures.

4.4 Measuring the robustness of community detection algorithms

We do not know the specific community structure of the karate network and the dolphin network, therefore an extension of the GN benchmark network is used [41]. GN benchmark is a set of synthetic networks with known community structures proposed by Girvan and Newman [16]. GN networks have been widely used as a benchmark for community detection algorithms. In this study, the GN network has 128 nodes, which are divided into 4 communities, and each community includes 32 nodes. Each node has an average z_{in} edges connecting it to members of the same community and z_{out} edges connecting it to members of other communities. Moreover, z_{in} and z_{out} are chosen to satisfy the total expected degree of a node $z_{\text{in}} + z_{\text{out}} = 16$. To illustrate the results more clearly, we set the GN3 network as an example. Because the community structure of the GN3 network (z_{out} =3) is quite obvious, there is no need to continue to enhance its community structures, but it is necessary to weaken its community structure to test the robustness of community detection algorithms. Hence we can use the null network of decreasing community structures to study the impact of community structures of different intensities on community detection algorithms.

As shown in Figure 10a, we use modularity Q to verify the performance of different community detection algorithms. When the property of the community structure is strong, we find that all algorithms have a good performance, except for KClique algorithm and LPLS algorithm. As the community structure weakens, the curves of InfoMap algorithm and label propagation algorithm have more fluctuations while the other algorithms are more smooth, so the robustness of these two algorithms are poor. When the property of the community structure is weak, the robustness of MultiLevel algorithm, CNM algorithm, DECD algorithm, LPLS algorithm and WalkTrap algorithm are better than that of GN algorithm and KClique algorithm. As shown in Figure 10b, we use the NMI values to verify the performance of different community detection algorithms. In the first two stages (community structure is strong and the process of weakening community structures), the NMI results are the same as the modularity Q results, as shown in Figure 10a. However, there is a great difference between the modularity Q and the NMI results when the community structure is weak, that is, the robustness of GN

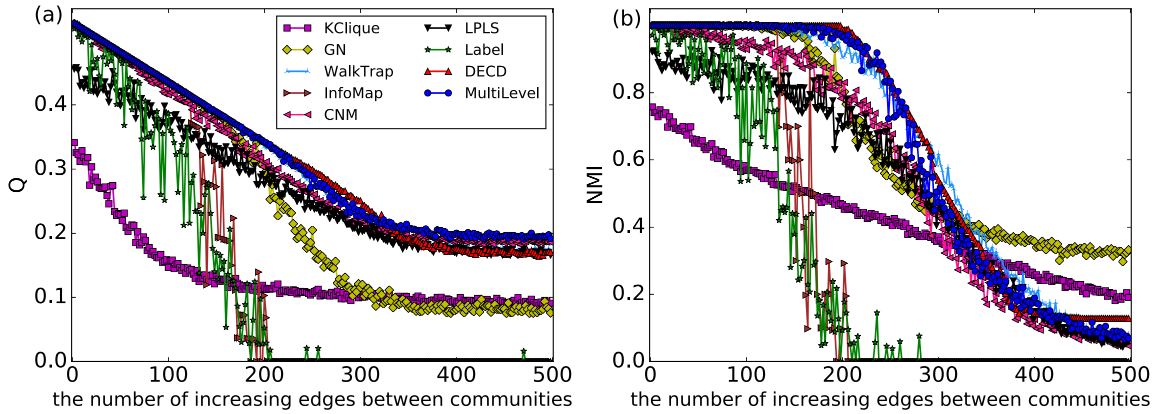

Fig. 10. The modularity Q and NMI results of the null network of decreasing community structures. The x-coordinate represents the number of increasing edges between communities, and the y-coordinate represents the two indexes. (a) The modularity index Q results, and (b) the NMI results. 10 typical modularity optimization algorithms (i.e., KClique [17], GN [16], MultiLevel [13], WalkTrap [15], InfoMap [18], CNM [14], LPLS [19], Label [20] and DECD [21]).

algorithm and KClique algorithm are better than that of other algorithms.

As shown in Figure 10, the modularity Q and NMI values of the various community detection algorithms decreased with the weakening of community structures. If community structure is strong, both the two indicators can measure the performance of all community detection algorithms effectively. When the community structure is weak, NMI can reflect the real community detection results, but the modularity Q can not. Our results indicate that the modularity statistic Q is not a suitable index for the network with weak community structures. Furthermore, our framework can test the robustness of any community detection algorithm very well. And our findings also suggest that when measuring the performance of community detection algorithms, we should not only pay attention to the situation that the community structure is strong, but also pay more attention to the situation when the community structure is weak.

5 Conclusion

In summary, we studied the relationship between the modularity index Q and network multi-lever micro-structures. We studied community detection results in artificial and real-life complex networks by constructing null networks. We maintained the micro-scale structures by constructing traditional $1k$–$3k$ null networks to study how micro-scale structures affect community properties. Experimental results demonstrate that the $1k$–$2k$ micro-structures (degree distribution and assortativity) are not enough to keep community properties, but $3k$ micro-scale structure is a significant micro-structure for maintaining the meso-scale characteristics of the original network.

Meanwhile, we maintained the community structure characteristics by constructing two novel null networks (the null network of rewiring edges within a community and the null network of rewiring edges between communities) to study the impact of the two types of edges on community structures. The results suggest that rewiring the two types of edges has a small impact on community structures, in the case of maintaining the community structure characteristics. Experimental results demonstrate that keeping community properties of the original network but not keeping its micro-scale structures are enough for community detection. We used the null network of rewiring edges within a community to study the impact of a single community structure on the entire network. The results suggest that the inner topology structures of every community have different roles in community properties of the entire network.

Furthermore, we changed community structure characteristics by constructing two novel null networks (increasing and decreasing community structures) to verify the performances of different community detection algorithms. We find that the performances of these detection algorithms have a significant difference when the edges in community structures have different intensities. Our results indicate that modularity Q is not a suitable index for the network with weak community structures. In conclusion, our findings can not only be used to test the robustness of different community detection algorithms, but also be helpful to uncover the correlation relationship of network structures between micro-scale and meso-scale. In considering future directions for research, we will expand the research framework from single-layer networks to multiplex networks, and further study the application of null networks in double-layer community detection.

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Author contribution statement

Wen-Kuo Cui, Yong-Jian Zhang and Xiao-Ke Xu designed research, Wen-Kuo Cui, Yong-Jian Zhang, Jing Xiao performed research, Wen-Kuo Cui, Ke-Ke Shang and Yong-Jian Zhang analyzed the data, and Wen-Kuo Cui, Ke-Ke Shang wrote the paper.

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