The Family of Assortativity Coefficients in Signed Social Networks

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***Abstract*— Signed networks are a special type of networks with both positive and negative edges, and the signs of links play a significant role in functional analysis and structural evolution. Because of the particularity of signed networks, the existing methods to measure their degree assortativity only rely on dividing the original network by link signs ignoring the signs of node degrees, which cannot measure the complicated degree mixing patterns. In this study, considering the complex types of link signs and node degree signs, we propose four new mixing patterns that have not been measured before for signed social networks and define a set of six signed assortativity measures based on traditional assortativity coefficients, to form a complete family of assortativity coefficients. The statistical significance of the family of assortativity coefficients is confirmed by comparing with null models, showing the assortativity significance profile (ASP) of different empirical signed networks. Besides, the relationship and distinction between the family of assortativity coefficients and classical network indexes, such as excess average degree and network embeddedness, are analyzed, revealing the endogenous complexity of signed social networks and the diversity of their assortativities. The proposed method enriches the assortativity diversity of social networks, which is beneficial to measure and analyze complex structure, function, and evolution of real-world social networks.**

***Index Terms*— Assortativity coefficient, assortativity significance profile (ASP), null model, signed network.**

I. Introduction

IN THE field of social networks, there are multiple mixing patterns according to different attributes of nodes or users [1]. Among them, the most widely studied one is degree assortativity, which has been defined to quantify the tendency of nodes to link to other nodes with a similar number of edges [2]. A network is assortative if high-degree nodes tend to connect to other high-degree nodes and low-degree nodes tend to connect to other low-degree nodes, whereas it is disassortative if high-degree nodes tend to connect to low-degree nodes [3]. Degree assortativity is a vital structural feature of social networks for researching the evolutionary, functional, and dynamic processes taking place in complex social systems, such as friend recommendation [4], synchronization processes [5], and epidemic spreading [6].

One of the main methods to characterize degree assortativity of complex networks is the degree assortativity coefficient proposed by Newman utilizing the Pearson correlation coefficient [2]. At present, the assortativity coefficient as a common statistic has been used in many studies of online social networks, and different degree mixing patterns have been found. For instance, the largest online social network in South Korea, Cyworld, is disassortative [7], while Facebook with more than 700 million nodes is assortative [8]. Besides, in the observation of Wealink, Hu and Wang [9] revealed the transition phenomenon of degree assortativity to disassortativity when an online social network is growing. Although Newman's assortativity coefficient can characterize the degree assortativity of real-life networks above, it cannot be applicable to the networks with directed or signed edges. Therefore, for the case of directed networks, Foster *et al.* [10] defined a set of four directed assortativity measures to characterize different degree–degree correlations. However, how to measure the degree assortativity of signed networks, such as Epinions [11] and Wiki-RFA [12], remains an open problem yet.

Signed social networks are a special type of social networks with both positive and negative edges [13], [14]. The positive edges represent positive relationships such as "friends" and "trust," which are indicated by the sign of "+," whereas the negative edges describe negative relationships such as "enemies" and "distrust," which are represented by the sign of "–." Usually, the number of positive/negative edges of a node is considered to be the positive/negative degree of this node. In the studies of the assortativity of signed networks, Szell and Thurner [15] found that the degree assortativities of positive (friends) and negative (enemies) subnetworks were different. Then, Rathore *et al.* [16] studied degree mixing patterns by considering negative edges and classified the nodes based on trustworthiness. Ciotti *et al.* [17] divided the signed network into the positive and negative subnetworks composed by the links with the same sign, revealing the assortativities of two subnetworks. However, in a signed network, each node has a positive degree and a negative degree, and the previous methods only separate the network through link signs and not consider the complexity of degree signs. Therefore, the case that link signs and degree signs are consistent has been explored, but few people try to measure other types of complex degree mixing patterns in signed social networks.

To solve this problem, the new measure of degree mixing patterns of signed networks is needed. First, more degree

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mixing patterns will be explored. Then, six signed assortativity measures are defined by modifying Newman's assortativity coefficient, composing the family of assortativity coefficients. In the experiment of four real-life signed social networks, the proposed measures of each network are compared with null models to estimate their statistical significance, and the assortativity significance profile (ASP) is shown. Finally, in comparison with other classical indexes, the results reveal that it is more effective to use the family of assortativity coefficients to characterize the degree mixing patterns of signed networks. Our method is based on the global structure rather than measuring certain nodes or edges, and we can compare the degree assortativity of multiple mixing patterns across networks of different sizes and types. The family enriches the assortativity diversity of social networks so that the characteristics of different types of degree mixing patterns can be measured and analyzed in the whole social network, which is beneficial to people's understanding of degree correlation of real-world social networks. Besides, it provides a new method for the statistical analysis of networks with different attitudes in the field of social science so that people can clearly understand the topology property of these networks, such as signed networks and trust networks [18].

II. Description of Empirical Network Data

In this study, we use four signed online social networks to analyze degree assortativity: Epinions, Bitcoinalpha, Wiki-RfA, and Slashdot, which can be obtained from the Stanford Network Analysis Project website [19]. It should be noted that we only use the sign information of four empirical networks to build undirected signed networks, discarding the direction and weight of original links.

Epinions is a who-trust-whom online social network of a general consumer review site Epinions [20]. Registered users of Epinions can declare their trust or distrust toward one another, based on the comments they post. The "trust" relationship between two users represents a positive edge, and conversely, the "distrust" relationship between two users represents a negative edge. The network consists of 131 828 nodes and 711 783 edges, among which 592 551 edges are positive edges accounting for 83.2%, and 119 232 edges are negative edges accounting for 16.8%.

Bitcoinalpha is a trusted network of people who trade using Bitcoin on the platform called Bitcoin Alpha [21]. Bitcoin users are anonymous, so it is necessary to maintain user reputation records in order to prevent transactions with fraudulent and risky users. Users of these platforms rate other users in a scale of -10 (total distrust) to $+10$ (total trust) in steps of 1. The sign of a user's rating for another user is the sign of the edge formed by the two users. The network consists of 3783 nodes and 14 124 edges, among which 12 759 edges are positive edges accounting for 90.3% and 1365 edges are negative edges accounting for 9.7%.

Wiki-RfA is a network of voting between Wikipedia members. To make a Wikipedia editor to be an administrator, a candidate or another community member must submit a Request for Adminship (RfA) [22]. Subsequently, any Wikipedia member can vote for support, neutrality, or opposition. The nodes of this network represent these members, and the positive and negative edges represent the votes supported and opposed, respectively. The network consists of 10 835 nodes and 171 800 edges, among which 132 954 edges are positive edges accounting for 77.4% and 38 846 edges are negative edges accounting for 22.6%.

Slashdot is a network formed by users tagging each other as friends or enemies on the website called Slashdot (www.slashdot.org), which is a technology-related news website known for its specific user community [20]. This network was obtained in February 2009 in which nodes represent users, and the positive/negative edges represent friends/enemies. The network consists of 82 144 nodes and 500 481 edges, among which 381 648 edges are positive edges accounting for 76.3% and 118 833 edges are negative edges accounting for 23.7%.

III. Familiy of Assortativity Coefficients

A. *Deficiency of Existing Statistics*

The traditional assortativity coefficient defined by Newman using the Pearson correlation coefficient is a common measure of degree assortativity [2]. It can be written as

$$r = \frac{M^{-1}\sum_i j_i k_i - \left[M^{-1}\sum_i \frac{1}{2}(j_i + k_i)\right]^2}{M^{-1}\sum_i \frac{1}{2}\left(j_i^2 + k_i^2\right) - \left[M^{-1}\sum_i \frac{1}{2}(j_i + k_i)\right]^2} \quad (1)$$

where M is the number of edges of the network and j_i and k_i are the degrees of the nodes at the ends of the ith edge, respectively. The range of r is from -1 to 1. When $r > 0$, the specified network is assortative; when $r < 0$, the specified network is disassortative; when $r = 0$, the specified network has no assortativity.

This statistic of r can only be applied to quantify the degree mixing pattern of an undirected network but not apply to directed networks or signed networks. Therefore, people are trying to look for new measures to reveal the assortativity properties of these special types of networks [10]. For signed networks, the existing method is to divide a network into two subnetworks as shown in Fig. 1 and then calculate the r values of positive and negative subnetworks, respectively [17].

Fig. 1(a) shows a small-scale signed network with seven nodes, where the blue solid line represents a positive link, the dotted red line indicates a negative link, and the numbers in the circle represents a positive degree and a negative degree, respectively. The signed network can be divided into a positive subnetwork [see Fig. 1(b)] and a negative subnetwork [see Fig. 1(c)] according to link signs. In the empirical network such as Epinions, two signed subnetworks can be extracted: the "trust" and "distrust" subnetworks in which all the links are positive or negative, respectively.

However, the degrees of nodes should also consider the effect of different signs. In Fig. 1(b), the sign of "-" in circle of each node represents that we have neglected negative node degree, and in Fig. 1(c), the sign of "+" in circle represents that we have neglected positive node degree. As far as we know, the current method in [17] just can describe the case that the link signs and degree signs are consistent, that is, the r-value of the positive subnetwork only characterize the

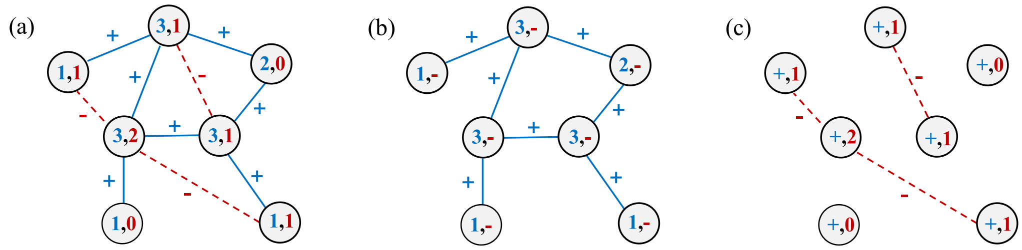

Fig. 1. Small-scale signed network with its positive and negative subnetworks. (a) Small-scale signed network, (b) its corresponding positive subnetwork, and (c) its corresponding negative subnetwork.

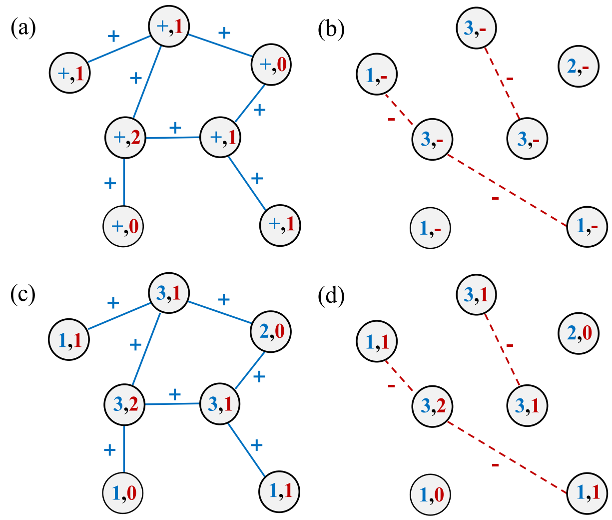

Fig. 2. Four new degree mixing patterns of signed networks.

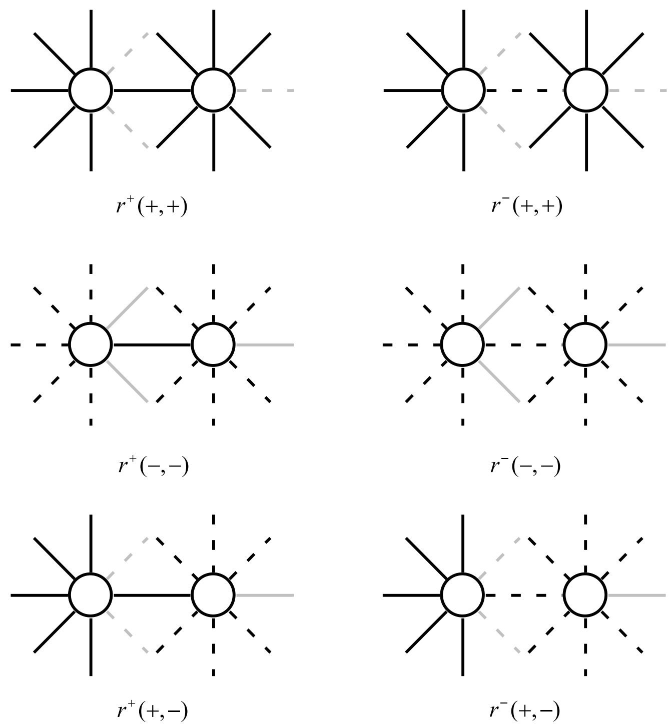

Fig. 3. Six degree–degree correlations and their measures in signed networks.

positive degree mixing pattern in which the nodes are connected by positive edges. Similarly, the negative subnetwork only characterizes the negative degree mixing pattern in which nodes are negatively connected. However, there are more complex degree mixing patterns that are shown in Fig. 2, such as negative degree mixing pattern in which nodes are connected by positive edges [see Fig. 2(a)] and positive degree mixing pattern in which nodes are negatively connected [see Fig. 2(b)]. Besides, each node has a positive degree and a negative degree. When the nodes are positively connected, their positive and negative degree mixing pattern is shown in Fig. 2(c). At the same time, when the nodes are negatively connected, the positive and negative degree mixing pattern is shown in Fig. 2(d). Therefore, the traditional methods for quantifying the degree mixing cannot fully reveal diverse assortativity patterns of a signed network.

B. Family of Assortativity Coefficients

For the above deficiency, new measures are needed to quantify complex mixing patterns in signed social networks. First, the two original patterns in Fig. 1(b) and (c) and four new mixing patterns in Fig. 2 are refined into six degree–degree correlations, and six assortativity measures are comprehensively designed in Fig. 3.

In Fig. 3, if there is a positive link between two nodes, the measure of positive–positive degree correlation [i.e., refined in Fig. 1(b)] is $r^+(+,+)$. If there is a positive link between two nodes, the measure of negative–negative degree correlation [i.e., refined in Fig. 2(a)] is $r^+(-,-)$. Similarly, if there is a positive link between two nodes, the measure of positive–negative degree correlation [i.e., refined in Fig. 2(c)] is $r^+(+,-)$. For the case of a negative link between two nodes, the measure of positive–positive degree correlation [i.e., refined in Fig. 2(b)] is $r^-(+,+)$, the measure of negative–negative degree correlation [i.e., refined by Fig. 1(c)] is $r^-(-,-)$, and the measure of positive–negative degree correlation [i.e., refined in Fig. 2(d)] is $r^-(+,-)$.

Then, we modify Newman's Pearson correlation coefficient to let $r^+(+,+)$, $r^-(+,+)$, $r^+(-,-)$, $r^-(-,-)$, $r^+(+,-)$, and $r^-(+,-)$ as

$$r^+(+,+) = \frac{M_+^{-1}\sum_i j_i^+k_i^+ - \left[M_+^{-1}\sum_i \frac{1}{2}\left(j_i^+ + k_i^+\right)\right]^2}{M_+^{-1}\sum_i \frac{1}{2}\left(\left(j_i^+\right)^2 + \left(k_i^+\right)^2\right) - \left[M_+^{-1}\sum_i \frac{1}{2}\left(j_i^+ + k_i^+\right)\right]^2} \quad (2)$$

$$r^{-}(+,+) = \frac{M_{-}^{-1}\sum_i j_i^{+}k_i^{+} - \left[M_{-}^{-1}\sum_i \frac{1}{2}\left(j_i^{+}+k_i^{+}\right)\right]^2}{M_{-}^{-1}\sum_i \frac{1}{2}\left(\left(j_i^{+}\right)^2+\left(k_i^{+}\right)^2\right) - \left[M_{-}^{-1}\sum_i \frac{1}{2}\left(j_i^{+}+k_i^{+}\right)\right]^2} \tag{3}$$

$$r^{+}(-,-) = \frac{M_{+}^{-1}\sum_i j_i^{-}k_i^{-} - \left[M_{+}^{-1}\sum_i \frac{1}{2}\left(j_i^{-}+k_i^{-}\right)\right]^2}{M_{+}^{-1}\sum_i \frac{1}{2}\left(\left(j_i^{-}\right)^2+\left(k_i^{-}\right)^2\right) - \left[M_{+}^{-1}\sum_i \frac{1}{2}\left(j_i^{-}+k_i^{-}\right)\right]^2} \tag{4}$$

$$r^{-}(-,-) = \frac{M_{-}^{-1}\sum_i j_i^{-}k_i^{-} - \left[M_{-}^{-1}\sum_i \frac{1}{2}\left(j_i^{-}+k_i^{-}\right)\right]^2}{M_{-}^{-1}\sum_i \frac{1}{2}\left(\left(j_i^{-}\right)^2+\left(k_i^{-}\right)^2\right) - \left[M_{-}^{-1}\sum_i \frac{1}{2}\left(j_i^{-}+k_i^{-}\right)\right]^2} \tag{5}$$

$$r^{+}(+,-) = \frac{M_{+}^{-1}\sum_i j_i^{+}k_i^{-} - \left[M_{+}^{-1}\sum_i \frac{1}{2}\left(j_i^{+}+k_i^{-}\right)\right]^2}{M_{+}^{-1}\sum_i \frac{1}{2}\left(\left(j_i^{+}\right)^2+\left(k_i^{-}\right)^2\right) - \left[M_{+}^{-1}\sum_i \frac{1}{2}\left(j_i^{+}+k_i^{-}\right)\right]^2} \tag{6}$$

$$r^{-}(+,-) = \frac{M_{-}^{-1}\sum_i j_i^{+}k_i^{-} - \left[M_{-}^{-1}\sum_i \frac{1}{2}\left(j_i^{+}+k_i^{-}\right)\right]^2}{M_{-}^{-1}\sum_i \frac{1}{2}\left(\left(j_i^{+}\right)^2+\left(k_i^{-}\right)^2\right) - \left[M_{-}^{-1}\sum_i \frac{1}{2}\left(j_i^{+}+k_i^{-}\right)\right]^2} \tag{7}$$

where M_{+} is the number of positive edges, M_{-} is the number of negative edges, and j_i^{+} and j_i^{-} are the degrees of positive and negative edges of one node of the ith edge, respectively, and k_i^{+} and k_i^{-} are the degrees of the other node. Each type of degree–degree correlation corresponds to a new measure. The six measures are combined to form the family of assortativity coefficients, which can quantify different degree mixing patterns and analyze the complex linking trend of signed networks more comprehensively.

IV. Statistical Characteristics of the Proposed Assortativity Family

A. Null Models of Signed Networks

When calculating relevant statistics to analyze the properties of empirical signed networks, because the absolute values of network statistics are dimensionless, the experimental results may not be inaccurate [23]. Hence, most empirical studies only pay attention to the relative value for a real-life network after comparing it with its corresponding null models [24]. A random network with some of the same properties as an empirical network is called a null model [25], [26].

In this research, we use two commonly used signed null models: sign randomized null model and full randomized null model. The sign randomized null model is that the link signs are randomized while keeping the positions of the edges fixed [27]. The full randomized null model is that both the signs and the positions of the edges are randomized [28]. By using the two null models, we can demonstrate the robustness and credibility of our proposed method.

B. Statistic Values of Assortativity Coefficients

1) Z-Score and P-Value: In order to prove the validity of the family of assortativity coefficients in signed networks, we use two statistics Z-score and P-value as the relative values based on the above two null models, respectively. In this study, Z-score is the difference between the assortativity measure of the real-world network and the mean of its null models divided by the standard deviation [10]. We assign each measure a statistical significance by its Z-score. For example, the statistical significance of $r^{+}(+,+)$ can be characterized by Z-score, that is

$$Z^{+}(+,+) = \frac{r^{+}(+,+) - \langle r_r^{+}(+,+)\rangle}{\sigma_r^{+}(+,+)} \tag{8}$$

where $r^{+}(+,+)$ is the result of the original network, $\langle r_r^{+}(+,+)\rangle$ is the mean of the corresponding null models, and $\sigma_r^{+}(+,+)$ is the standard deviation. The same goes for other measures. The P-value is the probability of occurrence of the sample observation when the original hypothesis is true. It can be used to estimate significance level, in which $P = 1 - \Phi(Z)$ and $\Phi(Z)$ is the standard normal distribution $\Phi(x) = \int_{-\infty}^{x}(1/\sqrt{2\pi})e^{-t^2/2}dt$. The smaller the P-value, the more significant the result. Generally, when $P < 0.05$, the measure is considered statistically significant.

In four empirical network experiments, we calculate the Z-score and P-value of six assortativity measures to estimate the statistical significance of the family of assortativity coefficients. The results are shown in Table I. Z_s quantifies the difference between the original network and its sign randomized null models, and Z_f quantifies the difference between the original networks and its full randomized null models. Especially, the values of P_s and P_f are both far less than 0.05, indicating that all six assortativity measures are statistically significant. The abovementioned results confirm that there are indeed six nontrivial degree mixing patterns in signed networks.

2) Assortativity Significance Profile: The value of Z-score is related to the size of the network under test [29]. In general, the more nodes the network has, the higher the Z-score is. In order to unify the evaluation results to the same scale, Z-score can be generally standardized by defining the significance profile (SP) [30]. ASP is actually a variation of the SP, which is the vector of Z-scores normalized to length 1 [10]. The profile can clearly compare the assortativity strength of each metric across different networks.

We can arrange $Z^{+}(+,+)$, $Z^{+}(+,-)$, $Z^{+}(-,-)$, $Z^{-}(+,+)$, $Z^{-}(+,-)$, and $Z^{-}(-,-)$ as a more general form of $Z(\alpha,\beta)$, and then, it is normalized as

$$\mathrm{ASP}(\alpha,\beta) = \frac{Z(\alpha,\beta)}{\left(\sum_{\alpha,\beta} Z(\alpha,\beta)^2\right)^{1/2}} \tag{9}$$

where $\mathrm{ASP}(\alpha,\beta)$ represents the normalized value of the Z-scores. This normalization does not change the relative magnitude of significance, and the value range is

TABLE I
STATISTICAL RESULTS OF SIX SIGNED ASSORTATIVITY MEASURES IN FOUR EMPIRICAL SOCIAL NETWORKS

Measures	Network	original network	sign randomized	Z_s	P_s	full randomized	Z_f	P_f
$r^+(+,+)$	Epinions	-0.052	-0.070 ± 0.001	35.000	0	-0.064 ± 0.001	19.209	0
	Bitcoinalpha	-0.153	-0.169 ± 0.001	10.870	0	-0.131 ± 0.004	-5.691	$< 10^{-3}$
	Wiki-RfA	-0.057	-0.071 ± 0.001	13.274	0	-0.063 ± 0.002	2.975	0.001
	Slashdot	-0.067	-0.073 ± 0.001	11.327	0	-0.036 ± 0.001	-44.143	0
$r^-(+,+)$	Epinions	-0.135	-0.068 ± 0.002	-29.485	$< 10^{-3}$	-0.066 ± 0.002	-28.886	$< 10^{-3}$
	Bitcoinalpha	-0.241	-0.168 ± 0.013	-5.634	$< 10^{-3}$	-0.118 ± 0.015	-8.092	$< 10^{-3}$
	Wiki-RfA	-0.116	-0.072 ± 0.004	-10.045	$< 10^{-3}$	-0.065 ± 0.004	-12.402	$< 10^{-3}$
	Slashdot	-0.052	-0.073 ± 0.001	49.711	0	-0.035 ± 0.002	7.981	0
$r^+(-,-)$	Epinions	0.021	-0.069 ± 0.001	128.714	0	-0.063 ± 0.001	105.500	0
	Bitcoinalpha	-0.021	-0.155 ± 0.007	18.608	0	-0.129 ± 0.005	21.827	0
	Wiki-RfA	-0.032	-0.070 ± 0.002	21.554	0	-0.061 ± 0.002	14.540	0
	Slashdot	0.007	-0.072 ± 0.001	90.849	0	-0.037 ± 0.001	50.242	0
$r^-(-,-)$	Epinions	-0.144	-0.068 ± 0.002	-35.468	$< 10^{-3}$	-0.064 ± 0.003	-30.248	$< 10^{-3}$
	Bitcoinalpha	-0.234	-0.157 ± 0.012	-6.266	$< 10^{-3}$	-0.111 ± 0.019	-6.525	$< 10^{-3}$
	Wiki-RfA	-0.083	-0.071 ± 0.004	-2.986	0.001	-0.064 ± 0.004	-4.957	$< 10^{-3}$
	Slashdot	-0.166	-0.072 ± 0.002	-59.507	0	-0.036 ± 0.002	-68.516	0
$r^+(+,-)$	Epinions	-0.191	-0.208 ± 0.001	28.667	0	-0.212 ± 0.001	45.171	0
	Bitcoinalpha	-0.228	-0.256 ± 0.003	10.108	0	-0.240 ± 0.002	7.727	0
	Wiki-RfA	-0.318	-0.302 ± 0.001	-12.019	$< 10^{-3}$	-0.284 ± 0.002	-16.246	$< 10^{-3}$
	Slashdot	-0.115	-0.136 ± 0.001	22.235	0	-0.123 ± 0.001	13.000	0
$r^-(+,-)$	Epinions	-0.170	-0.206 ± 0.001	45.500	0	-0.213 ± 0.0007	61.286	0
	Bitcoinalpha	-0.349	-0.245 ± 0.004	-25.538	$< 10^{-3}$	-0.232 ± 0.0054	-21.598	$< 10^{-3}$
	Wiki-RfA	-0.245	-0.296 ± 0.002	27.595	0	-0278 ± 0.0013	25.775	0
	Slashdot	-0.027	-0.132 ± 0.001	164.451	0	-0.121 ± 0.0005	191.060	0

from -1 to 1. ASP $>$ 0 indicates that the original network is more assortative compared with null models and ASP $<$ 0 indicates that the original network is disassortative. The ASP values of $Z^+(+,+)$, $Z^+(+,-)$, $Z^+(-,-)$, $Z^-(+,+)$, $Z^-(+,-)$, and $Z^-(-,-)$ reflect the relative strength of the different assortativities. The closer the value of ASP is to 1, the more assortative or disassortative the pattern is.

In this study, we use the values of ASP to normalize the Z-scores of six measures and compare various degree mixing patterns in signed networks. The ASP values of four empirical networks based on two null models are shown in Fig. 4, where the blue line is the result-based full randomized null models, and the red line is the result-based sign randomized null models. These profiles show four new assortativity measures that are obviously different from two previous assortativity measures $r^+(+,+)$ and $r^-(-,-)$. As shown in Fig. 4(a), the ASP of $Z^+(+,+)$ is positive, indicating the positive degree mixing pattern in which nodes are positively connected is assortative (i.e., popular nodes tend to befriend popular nodes). The value of ASP of $Z^-(-,-)$ is negative, indicating that the negative degree mixing pattern in which nodes are negatively connected is disassortative (i.e., unpopular nodes tend to befriend unpopular nodes). However, different from them, ASP($Z^-(+,+)$) is negative, indicating that the positive degree mixing pattern in which nodes are negatively connected is disassortative. ASP($Z^+(-,-)$) is positive, indicating that the negative degree mixing pattern in which nodes are positively connected is assortative.

Similar results can be seen in other networks, as shown in Fig. 4(b)–(d). In Fig. 4(b), in addition to the ASP($Z^+(+,+)$) based on the full randomized null model, other three mixed patterns have a similar performance to Epinions in Fig. 4(a). In Fig. 4(c), the ASP of four mixing patterns is exactly similar to Epinions. In Fig. 4(d), ASP($Z^+(+,+)$) and ASP($Z^-(+,+)$) have different results based on different null models, but ASP($Z^+(-,-)$) and ASP($Z^-(-,-)$) are similar to Epinions. Besides, the values of ASP of $Z^+(+,-)$ show different trends in each network, so does $Z^-(+,-)$. They are markedly different from the two previous assortativity measures. For the six ASP values, we can find that ASP($Z^+(-,-)$) is the largest in Fig. 4(a) and ASP($Z^-(+,-)$) is the largest in Fig. 4(b)–(d). The result indicates that the proposed measures, such as $Z^+(-,-)$ and $Z^-(+,-)$, have stronger degree assortativity characteristics in real-life signed networks and deserve further investigation.

Furthermore, each profile of ASP is divided into three areas filled with green, yellow, and pink in Fig. 4. The green areas including $Z^+(+,+)$ and $Z^-(+,+)$] depicts the positive degree and positive degree correlation, the yellow areas [including $Z^+(-,-)$ and $Z^-(-,-)$] depicts the negative degree and negative degree correlation, and the pink areas (including $Z^+(+,-)$ and $Z^-(+,-)$) depicts the positive degree and negative degree correlation. In Fig. 4(a)–(c), similar results can be observed in green and yellow areas; when the nodes are connected by a positive edge, the degree correlation is assortative; when the nodes are connected by a negative edge, the degree correlation is disassortative. The result indicates that high-degree nodes of the same sign tend to trust each other more, which satisfies the structural balance theory of signed networks [31]. However, the assortativity of the pink part is diverse, which needs further research.

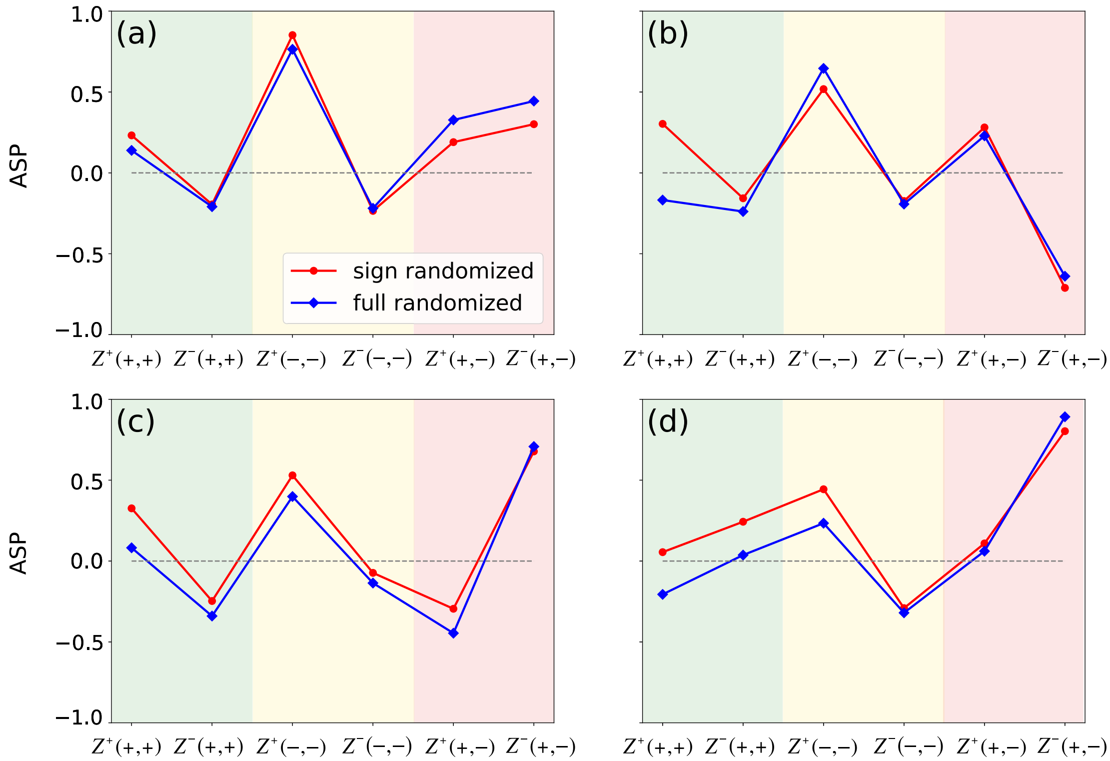

Fig. 4. ASP of six degree assortativity measures based on sign and full randomized null models. (a) Epinions. (b) Bitcoinalpha. (c) Wiki-RfA. (d) Slashdot.

V. Relationship Between the Family of Assortativity Coefficients and Classical Network Indexes

In the study of empirical signed networks, there are two other classical ways to measure network degree properties: excess average degree [32] and embeddedness index [33]. In the following experiment, we selected two typical networks, Epinions and Wiki-RfA, to explore the relationships and distinctions between the proposed assortativity coefficients and the two classical indicators.

A. Relationship Between the Family of Assortativity Coefficients and Excess Average Degree

In order to study the relationship between the family of assortativity coefficients and the statistic of excess average degree, the empirical signed network is divided into positive and negative subnetworks, respectively. We try to observe the relative trend of excess average degree between the original subnetwork and null models and compare the results with assortativity property quantized by the proposed assortativity coefficients.

The statistic of excess average degree is a method to calculate the average degree of the nearest neighbors of nodes with a degree k, denoted as $K_{nn}(k)$ [32]. Suppose that the degree of the k_i neighbor nodes of node i is k_{i_j}, $j = 1, 2, \ldots, k_i$. The average degree $\langle K_{nn}\rangle_i$ of k_i neighbor nodes of node i is as follows:

$$\langle K_{nn}\rangle_i = \frac{1}{k_i}\sum_{j=1}^{k_i} k_{i_j}. \tag{10}$$

Assuming that the nodes with degree k in a network are $\upsilon_1, \upsilon_2, \ldots, \upsilon_k$, the excess average degree $K_{nn}(k)$ of the nodes with degree k can be calculated by the following:

$$K_{nn}(k) = \frac{1}{i_k}\sum_{i=1}^{i_k} \langle k_{nn}\rangle_{\upsilon_i}. \tag{11}$$

If $K_{nn}(k)$ is an increasing function of k on average, it indicates that high-degree nodes tend to connect to other high-degree nodes, which means that the network is assortative. Conversely, if $K_{nn}(k)$ is a decreasing function of k, it indicates that high-degree nodes tend to connect to low-degree nodes, so the network is disassortative. If there are no degree correlations in a network, $K_{nn}(k)$ does not have a monotonic trend with respect to k.

In Fig. 4(a) and (c), the assortativity of Epinions and Wiki-RfA has been depicted by ASP. The tendencies quantified by $r^+(+,+)$ are both assortative and quantified by $r^-(-,-)$ are both disassortative. The assortativity of the mixing pattern characterized by $r^+(+,+)$ corresponds to the assortativity of the positive subnetwork, and the mixing pattern characterized by $r^-(-,-)$ corresponds to the negative subnetwork. As shown in Fig. 5(a), $K_{nn}(k)$ of positive subnetwork in Epinions has an increasing behavior as a function of k compared with two null models, indicating that it is assortative. In Fig. 5(b), $K_{nn}(k)$ for the negative subnetwork of Epinions has a decreasing behavior as a function of k, indicating that it is disassortative. The abovementioned results are consistent with the results of $r^+(+,+)$ and $r^-(-,-)$ for Epinions. However, in the positive and negative subnetworks of Wiki-RfA, as shown in Fig. 5(c) and (d), the varying trend is not obvious as

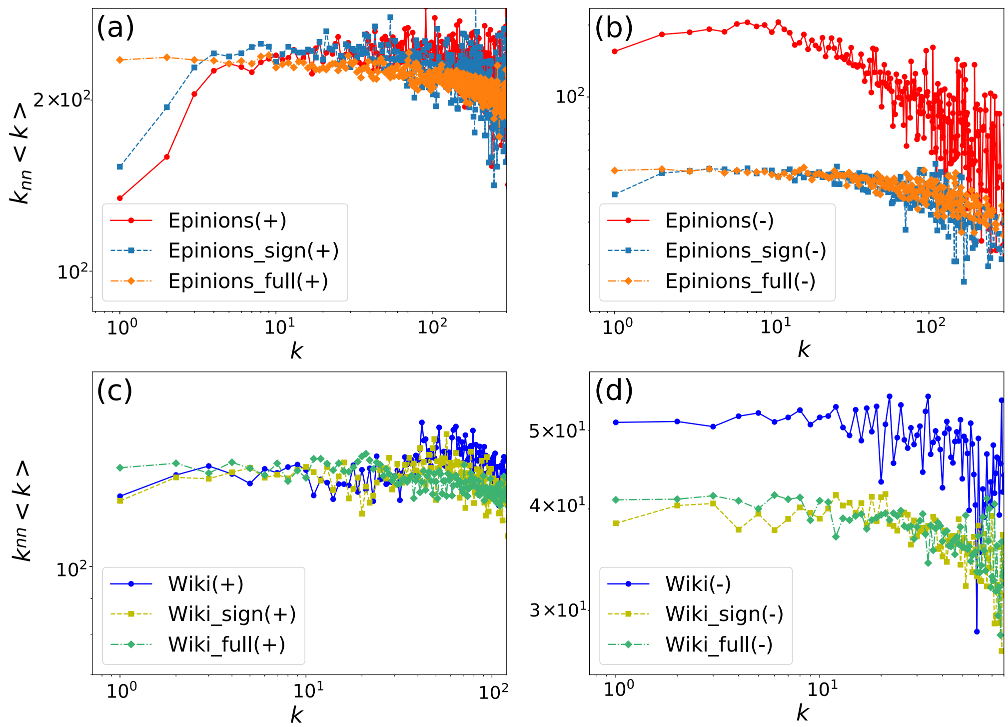

Fig. 5. $K_{nn}(k)$ values of positive and negative subnetworks. (a) Positive subnetwork of Epinions. (b) Negative subnetwork of Epinions. (c) Positive subnetwork of Wiki-RfA. (d) Negative subnetwork of Wiki-RfA.

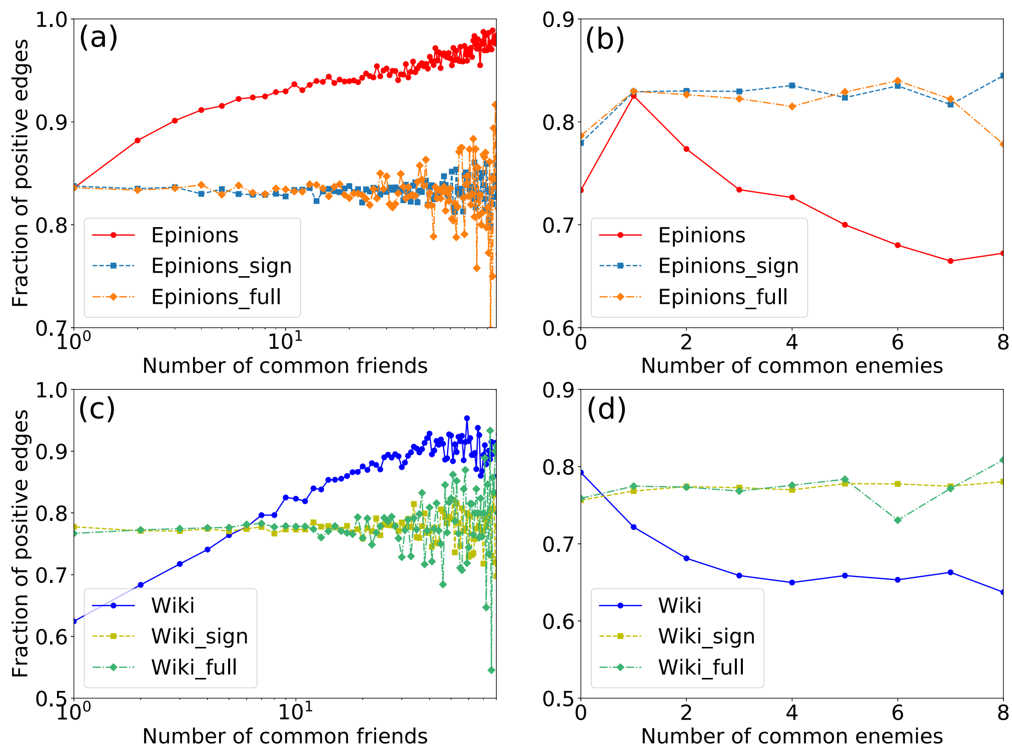

Fig. 6. Embeddedness of positive and negative subnetworks. (a) Positive subnetwork of Epinions. (b) Negative subnetwork of Epinions. (c) Positive subnetwork of Wiki-RfA. (d) Negative subnetwork of Wiki-RfA.

a function of k compared with the null models, and the assortativity pattern cannot be judged by excess average degree. However, the results of $r^{+}(+,+)$ and $r^{-}(-,-)$ clearly reveal that the positive subnetwork is assortative and the negative subnetwork is disassortative in Wiki-RfA. The phenomenon reveals that the excess average degree can only judge individual networks by changing trends, while our proposed measures can quantify the complex assortativity of any signed social network.

B. Relationship Between the Family of Assortativity Coefficients and Embeddedness

In order to study the relationship between the family of assortativity coefficients and the statistic of embeddedness, we explore the probability that an edge is positive as a function of embeddedness. The experiment results on two sub-networks are compared with assortativity properties quantized by $r^+(+,+)$ and $r^+(-,-)$.

In a social network, the number of common neighbors between two users is called the embeddedness property of them [33]. If two users have more common neighbors, they are more embedded. According to the structural balance theory [31], the more embedded two nodes are, the higher probability that there is a positive edge between them is [20].

In Fig. 4(a) and (c), the tendencies are quantified by $r^+(+,+)$ and $r^+(-,-)$ are both assortative in Epinions and Wiki-RfA. The assortativity of the mixing pattern characterized by $r^+(+,+)$ corresponds to the fraction of positive edges in the case of positive embeddedness (that is, the greater the number of common friends), and the mixing pattern characterized by $r^+(-,-)$ corresponds to the possibility of positive edges in the case of negative embeddedness (that is, the greater the number of common enemies). Fig. 6(a) and (c) shows that with the increase of the number of common friends, the fraction of positive edges between two nodes increases significantly compared with two null models. The result is consistent with $r^+(+,+)$, indicating that the nodes with high positive degree more tend to be connected. Meanwhile, as shown in Fig. 6(b) and (d), with the increase of the number of common enemies, the fraction of the positive edges has a negative trend in both Epinions and Wiki, which is different from that of $r^+(-,-)$. The result suggests that the probability of the existence of positive edges as increasing embeddedness only reveals the relationship between two nodes. However, the family of assortativity coefficients (including $r^+(+,+)$ and $r^+(-,-)$) is to characterize the assortativity between all the nodes and can depict the link tendency of high-degree nodes in an entire signed network more clearly.

VI. Conclusion

In summary, we proposed a novel method for measuring complex degree assortativity of signed social networks. First, the four novel degree mixing patterns were proposed, and a set of six signed assortativity measures were defined, forming the family of assortativity coefficients in signed social networks. Then, based on two kinds of null models, we proved that nontrivial degree mixing patterns do exist in real-life signed networks. By showing the ASP of four empirical signed social networks, the results revealed that the four new assortativity measures are obviously different from the two classical assortativity measures ($r^+(+,+)$ and $r^-(-,-)$), and the novel measures also have stronger degree assortativity characteristics. Furthermore, compared with other classical network indexes such as excess average degree and embeddedness index, our results confirmed the accuracy and effectiveness of the family of assortativity coefficients.

The six measures we defined consider only link signs but also node degree signs of signed networks, and different types of degree mixing patterns in a real-life network can be measured and analyzed. The proposed method can be used not only in online social networks but also in other types of signed networks, such as biological networks and information networks. In future, we will consider the trust formation of a node or an edge to further analyze the special phenomena of global mixing pattern. Also, our measures can also be applied to analyze trust networks, such as topology, dynamic behavior, and trust hole detection [34]. Moreover, because it can more comprehensively quantify the endogenous complexity of interconnection between nodes, it may play a significant role in the tasks of data mining for signed networks, such as sign prediction, link prediction, and community detection.

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